Roll No.							

25-MA-45

M.Sc. IV SEMESTER [MAIN/ATKT] EXAMINATION MAY - JUNE 2025

MATHEMATICS

Paper - V

[Analytic Number Theory - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 Prove that a Dirichlet series $\Sigma f(n)\, n^{-s}$ converges uniformly on every compact subset lying interior to the half plane of convergence $\Gamma > \Gamma c$

OR

Prove that if the series $\Sigma f(n)\, n^{-s}$ converges for $s = \sigma_0 + i\, t_0$ then it also converges for all s with $\sigma > \sigma_0$. If it diverges for $s = \sigma_0 + i\, t_0$ then it diverges for all s with $\sigma < \sigma_0$

Q. 2 State and prove an integral formula for the coefficients of a Dirichlet series.

OR

If $\sigma > 1$ then prove the following formulas -

a) $$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| \zeta^{(k)}(\sigma + i\,t) \right|^2 dt = \zeta^{(2k)}(2\sigma)$$

b) $$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| \zeta(\sigma + i\,t) \right|^{-2} dt = \frac{\zeta(2\sigma)}{\zeta(4\sigma)}$$

c) $$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| \zeta(\sigma + i\,t) \right|^{4} dt = \frac{\zeta^4(2\sigma)}{\zeta(4\sigma)}$$

P.T.O.

Q. 3 Prove that for $\sigma > 1$,

$$\Gamma(s)\,\zeta(s, a) = \int_0^\infty \frac{x^{s-1} e^{-ax}}{1 - e^{-x}}\, dx$$

also show that for a = 1

$$\Gamma(s)\,\zeta(s) = \int_0^\infty \frac{x^{s-1} e^{-x}}{1 - e^{-x}}\, dx$$

OR

If $0 < a \leq 1$ then show that

$$I(s, a) = \frac{1}{2\pi i} \int_c \frac{z^{s-1} e^{az}}{1 - e^{z}}\, dz \text{ is an entire function of s.}$$

and also deduce that $\zeta(s, a) = \Gamma(1 - s)\, I(s, a)$, if $\sigma > 1$.

Q. 4 State and prove Hurwitz's formula.

OR

State and prove a functional equation for the Hurwitz zeta function.

Q. 5 a) Explain Bernoulli polynomial with example.

b) Show that Bernoulli polynomials $B_n(x)$ satisfy the difference equation $B_n(x + 1) - B_n(x) = n\, x^{n-1}$ if $n \geq 1$, Also deduce that $B_n(0) = B_n(1)$ if $n \geq 2$.

OR

a) If $n \geq 0$, then show that $\zeta(-n) = -\frac{B_{n+1}}{n+1}$

b) Explain Bernoulli numbers with example. Also compute the Bernoulli numbers for n = 6, 7, 8, 9, 10