Roll No.							

25-MA-44

M.Sc. IV SEMESTER [MAIN/ATKT] EXAMINATION
MAY - JUNE 2025

MATHEMATICS

Paper - IV

[Operations Research - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 a) Prove that a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that **(07 marks)**

$$\sum_{i=1}^{m} O_i = \sum_{j=1}^{n} b_j = \lambda$$

b) Given $x_{13} = 50$ units, $x_{14} = 20$ units, $x_{21} = 55$ units, $x_{31} = 30$ units, $x_{32} = 35$ units, $x_{34} = 25$ units. Is it an optimal solution to the transportation problem - **(08 marks)**

					Available Units
	6	1	9	3	70
	11	5	2	8	55
	10	12	4	7	90
Required Units	85	35	50	45	

If not, modify it to obtain a better feasible solution.

OR

a) Find the basic feasible solution in the following transporting problem by -

i) Northwest corner rule.

ii) Least cost method. **(08 marks)**

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

P.T.O.

b) Write steps of Vogel's approximation method to find initial solution of any transportation problem. **(07 marks)**

Q. 2 Consider the following transportation problem - **(15 marks)**

Factory	Godowns 1	2	3	4	5	6	Stock Available
A	7	5	7	7	5	3	60
B	9	11	6	11	-	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

If is not possible to transport any quantity from factory B to Godown 5.

Determine -

i) Initial solution by Vogel's approximation method.

ii) Optimum Basic feasible solution.

iii) Is the optimum solution unique ?

If not, find the alternative optimum basic feasible solution.

OR

a) Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows - **(08 marks)**

Person		Job 1	2	3	4	5
	A	8	4	2	6	1
	B	0	9	5	5	4
Person	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

b) How would you deal with the assignment problem, where **(07 marks)**

i) The objective function is to be maximized ?

ii) Some assignments are prohibited ?

Cont. . . .

Q. 3 A small project consists of seven activities for which the relevant data are given below - **(15 marks)**

Activity	Preceding Activities	Duration
A	-	4
B	-	7
C	-	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

i) Draw the Network diagram and find project duration.

ii) Calculate total float for each activities.

iii) Draw the time scaled diagram.

OR

a) Explain the following terms used in PERT - **(07 marks)**

i) Pessimistic time.

ii) Optimistic time.

iii) Most likely time.

b) Write the steps used in Monto-Carlo simulation. **(08 marks)**

Q. 4 a) A project schedule consists of five activities and the duration of these activities is non deterministic with the following probability distribution - **(10 marks)**

Activity	Days	Probability
1-2	1	0.2
	4	0.5
	8	0.3
1-3	2	0.3
	4	0.5
	7	0.2
2-4	2	0.3
	4	0.3
	6	0.4

P.T.O.

3-4	3	0.3
	6	0.4
	8	0.3
4-5	2	0.2
	3	0.2
	4	0.6

Simulate the duration of the project 10 times and estimate the chances of the various paths to be critical. What is the average duration of the project.

b) Indicate any four shortcoming of taking a simulation approach to solve an O.R. problem. **(05 marks)**

OR

A project consists of 7 activities. The time for performance of each of the activities is a random variable with the respective probability distribution as given below - **(15 marks)**

Activity	Immediate Predecessor	Time (in days) and its probability				
A	-	3	4	5		
		0.20	0.60	0.20		
B	-	4	5	6	7	8
		0.10	0.30	0.30	0.20	0.10
C	A	1	3	5		
		0.1	0.75	0.10		
D	B, C	4	5			
		0.80	0.20			
E	D	3	4	5	6	
		0.10	0.30	0.30	0.30	
F	D	5	7			
		0.20	0.80			
G	E, F	2	3			
		0.50	0.50			

i) Draw the network diagram and identify the critical path using the expected activity times.

ii) Simulate the project using random numbers to determine the activity times. Find the critical path and the project duration.

iii) Repeat the simulation four more times. Is the same path critical in all the simulation ?

Cont. . . .

Q. 5 a) Let (a_i , j) be the m x n payoff matrix for a two person zero sum game. If $\underline{V}$ denotes the maximum value and $\overline{V}$ the minimax value of the game, then $\overline{V} \geq \underline{V}$. That is **(07 marks)**

$$\min_{1 \leq j \leq n} \left[\max_{1 \leq i \leq m} \{a_{ij}\} \right] \geq \max_{1 \leq i \leq m} \left[\min_{1 \leq i \leq n} \{a_{ij}\} \right]$$

b) Solve the following Non linear programming problem - **(08 marks)**

Maximize $z = 7x_1^2 + 6\,x_1 + 5x_2^2$

Subject to

$$x_1 + 2x_2 \leq 10$$

$$x_1 - 3x_2 \leq 9$$

$$x_1 , x_2 \geq 0$$

OR

a) For any 2 x 2 game two person zero sum without any saddle point having the pay off matrix for player A **(08 marks)**

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{array}$$

The optimum mixed strategies

$$S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix} \quad , S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$$

are determined by -

$$\frac{p_1}{p_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}} \quad \frac{q_1}{q_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$$

where $p_1 + p_2 = 1$ and $q_1 + q_2 = 1$. The value of game is given by -

$$V = \frac{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}{a_{11} + a_{22} - (a_{12} + a_{21})}$$

b) Explain the graphical method of solving 2 x n game. **(07 marks)**

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