Roll No.							

25-ST-24

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION MAY - JUNE 2025

STATISTICS

Paper - IV

[Statistical Inference - I]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 Let T_1 and T_2 be unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively and $\rho = \rho_\theta$ be the correlation coefficient between them. Then show that

$$\sqrt{e_1 e_2} - \sqrt{(1-e_1)(1-e_2)} \leq \rho \leq \sqrt{e_1 e_2} + \sqrt{(1-e_1)(1-e_2)}$$

OR

Discuss sufficient condition for consistency.

Q. 2 State and prove Cramer Rao inequality and write regularity conditions.

OR

a) Prove that under certain general conditions of regularity to be stated clearly the mean square deviation $E(\hat{\theta} - \theta)^2$ of an estimator $\hat{\theta}$ of parameter θ can never fall below a positive limit depending only on the density function $f(x, \theta)$, the size of the sample and the bias of the estimate.

b) State and prove Rao-Blackwell theorem.

Q. 3 a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the uniform distribution with p.d.f. -

$$f(x,\theta) \begin{cases} \dfrac{1}{(\theta)} & , \quad 0 < x < \infty \quad , \theta > 0 \\ 0 & , \quad \text{else wise} \end{cases}$$

obtain the maximum likelihood estimator for θ.

b) Obtain the MLEs for α and β for the rectangular population.

P.T.O.

OR

Obtain 100 $(1 - \alpha)$ % confidence intervals for the parameters (a) θ and (b) σ^2 of the normal distribution.

$$f(x, \theta; \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left\{ \frac{-1}{2} \left(\frac{x - \theta}{\sigma} \right)^2 \right\}, \quad -\infty < x < \infty$$

Q. 4 State and prove Neyman Pearson Lemma.

OR

Given the frequency function

$$f(x, \theta) \begin{cases} \dfrac{1}{(\theta)} & , \quad 0 \le x \le \theta \\ 0 & , \quad \text{otherwise} \end{cases}$$

and that you are testing the null hypothesis. $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value of x. What would be the sizes of the type I and type II errors, if you choose the interval (i) $0.5 \le x$ (ii) $1 \le x \le 1.5$ as the critical regions ? Also obtain the power function of the test.

Q. 5 Describe likelihood Ratio Test.

OR

Let $x_1, x_2, \ldots, x_n$ be a random sample from a $N(\mu, \theta)$ where θ is the unknown variance and μ is known

Obtain a likelihood ratio test for testing a simple $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$

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