Roll No.							

25-MA-24

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION MAY - JUNE 2025

MATHEMATICS

Paper - IV

[Complex Analysis]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt any **2** Part from each Question.
Each question carries **15 marks.**

Q. 1 a) State and prove Hurwitz theorem.

b) State and prove Riemann mapping theorem.

c) Prove that C (G, Ω) is a complete metric space.

Q. 2 a) Prove that

$$\Gamma z = \lim_{n \to \infty} \frac{n\,!\,n^{z}}{z\,(z+1)\,....\,(z+n)} \quad , \text{for } z \neq 0, -1, ...$$

b) State and prove Runge's theorem.

c) Prove that

$$\zeta(z)\,\Gamma z \int_0^{\infty} = (e^{t} - 1)^{-1}\, t^{z-1}\, dt\,, \text{ for Re } z > 1$$

Q. 3 a) State and prove Hadmard three circle theorem.

b) If f(z) be an entire function with $f(0) \neq 0$ also $r_1, r_2, ..., r_n$ be the moduli of zeros $z_1, z_2, ..., z_n$ of f(z) res p arranged as non decreasing sequence multiple zero being repeated, then $R^n \mid f(0) \mid < M(R)\, r_1, r_2, ... r_n$ where $r_n < R < r_{n+1}$ and M(R) is maximum modulus of f(z)

c) State and prove Hadmard Factorization theorem.

Q. 4 a) Show that the series $\sum_{n=1}^{\infty} \frac{z^n}{2^{n+1}}$ and $\sum_{n=1}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$

are analytic continuation to each other.

P.T.O.

b) State and prove Monodromy theorem.

c) Prove that there can not be more than one analytic continuation of a function into the same domain.

Q. 5 a) State and prove Kobe's 1/4 theorem.

b) State and prove Great Picard theorem.

c) If f be an entire function that omits two values then f is constant.

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