Roll No.							

25-MA-23

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION MAY - JUNE 2025

MATHEMATICS

Paper - III

[Topology - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 a) Let Y be a subspace of a topological space X. Then prove that Y is compact if and only if every covering of Y by the sets open in X contains a finite sub collection covering Y. **(10 Marks)**

b) Prove that the image of a compact space under a continuous map is compact. **(05 Marks)**

OR

Let X be a metrizable space. then prove that the following statements are equivalent - **(15 Marks)**

i) X is compact.

ii) X is limit point compact.

iii) X is sequentially compact.

Q. 2 a) Prove that every metrizable space is normal. **(10 Marks)**

b) Prove that a closed subspace of a normal space is normal. **(05 Marks)**

OR

State and Prove Urysohn's Lemma. **(15 Marks)**

Q. 3 a) Prove that the product of two second countable spaces is second countable. **(10 Marks)**

b) Prove that the product of two T_2 - spaces is a T_2-Space. **(05 Marks)**

OR

Prove that the projection maps are continuous and open. **(15 Marks)**

P.T.O.

Q. 4 a) Prove that the arbitrary intersection of filters is a filter. **(10 Marks)**

b) Prove that every limit point of a filter is a cluster point of the filter. **(05 Marks)**

OR

Prove that every filter is contained in an ultra filter. **(15 Marks)**

Q. 5 Define Homotopy relation ' $\simeq$ ' with example and show that the relation ' $\simeq$ ' is an equivalence relation **(15 Marks)**

OR

a) Prove that the map $\hat{\alpha}$ is a group isomorphism. **(05 Marks)**

b) If $p : E \rightarrow B$ and $p' : E' \rightarrow B'$ are covering maps. Then prove that $p \times p' : E \times E' \rightarrow B \times B'$ is a covering map. **(10 Marks)**

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