Roll No.							

25-MA-22

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
MAY - JUNE 2025

MATHEMATICS

Paper - II

[Lebesgue Measure and Integration]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 Attempt **any two** from the following -

a) Prove that the outer measure of an interval equals its length.

b) Show that the union of a countable collection of measurable sets is measurable.

c) Let f and g be measurable function on E that are finite a.e. on E, then for any α , β in R, prove that $\alpha f + \beta g$ and f g are measurable function on E.

Q. 2 a) Let φ and ψ be simple functions which vanish outside a set of finite measure then show that -

i) $\int (\alpha\varphi + \beta\psi) = \alpha \int \varphi + \beta \int \psi \quad \forall\, \alpha , \beta \in \mathbb{R}$.

ii) If $\varphi \geq \psi$ a.e., then $\int \varphi \geq \int \psi$

b) State and prove Lebesgue Bounded convergence theorem.

OR

a) Let {fn} be a sequence of measurable functions such that $| f_n | \leq g$, where g is integrable, and let $\lim f_n = f$ a.e. then prove that f is integrable and $\lim \int f_n \, dx = \int f \, dx$

b) Let $\{f_n\}$ be a sequence of non negative measurable functions and let $f = \sum_{n=1}^{\infty} f_n$. Then prove that $\int f = \sum_{n=1}^{\infty} \int f_n$

Q. 3 Attempt **any two** from the following -

a) A bounded monotone function is a function of bounded variation, prove it.

b) Let $f \in BV[a, b]$, then prove that $f(b) - f(a) = P - N$ and $T = P + N$ where T, P and N are total variation, positive variation and negative variation respectively. All variation being on the finite interval [a, b]

c) Let f be a bounded and measurable function defined on [a, b], if

$$F(x) = \int_a^x f(t)\,dt + F(a) \text{ then prove that}$$

$$F(x) = f(x) \text{ a.e. in } [a, b]$$

Q. 4 a) State and prove Jenesen's inequality.

b) State and prove Holder's inequality for $1 < p < \infty$

OR

Prove that L^P - spaces are complete

Q. 5 Attempt **any two** from the following -

a) If $\{f_n\}$ is a sequence of measurable functions which is convergence in measure of f, then prove that it convergence in measure to every function g, which is equivalent to the function f.

b) If $\{f_n\}$ is a sequence of measurable functions which is fundamental in measure, then prove that there exists a measurable function f such that $f_n \to f$ in measure.

c) If $f_n \to f$ a.u. then prove that $f_n \to f$ a.e.

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