Roll No.							

25-MA-21

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION MAY - JUNE 2025

MATHEMATICS

Paper - I

[Advanced Abstract Algebra - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt all five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 a) Define R-module with example and show that the polynomial ring R[x] over a ring R is an R-module.

b) Let $(N_i)_{i \in \wedge}$ be a family of R-sub modules of an R-module M. Then prove that $\cap_{i \in \wedge} N_i$ is also an R-sub module.

OR

a) If f : M → N is an R - homomorphism of an R-module M to an R-module N, then show that

i) Ker f is an R-sub module of M.

ii) Imf is an R-sub module of N.

b) Let M be an R-module. Then prove that Hom_R (M, M) is a subring of Hom (M, M)

Q. 2 a) Let V be a non zero finitely generated vector space over a field F. Then prove that V admits a finite basis.

b) Let M be a finitely generated free module over a commutative ring R. Then prove that all bases of M have the same number of elements.

OR

a) Let R be a ring with unity, and let M be an R-module. Then prove that M is simple if and only if M ≠ (0) and M is generated by any $0 \neq x \in M$

b) Find the rank f the Linear Mapping $\phi : R^4 \rightarrow R^3$ where

ϕ (a, b, c, d) = (a + 2b – c + d, – 3 a + b + 2 c – d, – 3 a + 8 b + c + d)

P.T.O.

Q. 3 Prove that for an R-module M, the following are equivalent -

i) M is artinian.

ii) Every quotient module M is finitely co generated.

iii) Every non empty set S of sub modules of M has a minimal element.

OR

a) Prove that every submodule and every homomorphic image of a noetherian module is noetherian.

b) Let R be a noetherian ring. Then prove that the polynomial ring R[x] is also a noetherian ring.

Q. 4 State and prove uniqueness of a decomposition of a finitely generated module over a principal ideal domain.

OR

a) Let U be a uniform module over a commutative noetherian ring R. Then prove that U contains a submodule isomorphic to R/P for precisely one prime ideal P.

b) Find the abelian group generated by $\{x_1, x_2, x_3\}$ subject to

$5x_1 + 9x_2 + 5x_3 = 0$

$2x_1 + 4x_2 + 2x_3 = 0$

$x_1 + x_2 - 3x_3 = 0$

Q. 5 Find rational canonical form of the matrix A where -

$$A = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & -2 \end{pmatrix}$$

OR

Let the linear transformation $T \in A_F(v)$ be nilpotent then prove that

$\alpha_0 + \alpha_1 T + \ldots + \alpha_m T^m$, where $\alpha_i \in F$, $0 \le i \le m$, is invertible if $\alpha_0 \neq 0$.

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